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**PUBLIC INVESTMENT AND HIGHER EDUCATION
INEQUALITY**

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Public investment and higher education inequality*

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Abstract

Empirical results show that children from high income households achieve higher levels of education and are more likely to be enrolled in post compulsory school. Theoretical findings fail to answer clearly whether greater public investment in the higher education system effectively decreases the inequality between the educational attainment of rich and poor children. We show that if the child receives a monetary transfer from his parents and allocates it between private consumption and investment in private additional education, then a further public investment decreases the educational gap. This result holds under the assumptions of both substitutability and complementarity between private and public education.

Keywords: Higher education inequality; Public education; Altruism

JEL classification: H31; H52; I21; J24

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1 Introduction

The distribution of education across individuals has been one of the most studied topics during the last two decades. Empirical and theoretical evidence confirms that children whose parents are characterized by high level of income and high human capital are more likely to be enrolled at university and have better higher education attainments (see, e.g., Haveman and Wolfe, 1995; Woessmann, 2004; Blau, 1999; Shea, 2000; Cameron and Heckman, 2001; Plug and Vijverberg, 2001, 2003; Taubman, 2001; Blanden, Gregg and Machin, 2003; Blandel and Machin, 2004; Callender, 2003; Chevalier and Lanot, 2002; De Fraja, 2002; Zanchi and Oliveira, 2003). Despite the fact the higher education inequality has been largely confirmed in several countries, there are many doubts about the existence of a policy for equality. In particular, there still exist concerns about the real effect of a policy increasing public investment.

This paper shows that increasing public investment in the higher education system is an effective policy to reduce the educational gap between students from high and low income households. We develop a theoretical model in which the educational attainment by the child is to be interpreted as human capital: it depends on public schooling and investment in private education by abstracting from parental education, students' ability and the effort they exert while at school. Private education includes time and private goods which increase the individual human capital; therefore the student is supposed to receive both public and private education. We assume that households are composed of one altruistic parent and his child; the only way the parent can finance the educational process of his child is to leave a monetary transfer. We consider this transfer as a sort of bequest. The decision-makers are assumed to be two: the parent and the child. The parent is the first mover and decides how to allocate his income between the bequest and his own current consumption. In the successive period the child receives public education and allocates his bequest between consumption good and investment in private education.

The assumption that the investment in private education is decided by the child differentiates our paper from the previous literature (Becker and Tomes 1986 and Nordblom 2003). We are aware that a model in which the private investment is chosen by the parent is appropriate for the compulsory school but it does not properly represent the educational process working at the university level. The parents are often not able to figure out the kind of educational support the child attending university really needs to improve his level of education. This difficulty could arise from the weak interaction between parents and academic staff (as professors) characterizing the university system. In some countries (such as Italy and the UK) the compulsory school guarantees frequent meetings between parents and teachers, but these do not take place once children go to university. Furthermore, the child's age also matters. The student attending compulsory school is quite young and needs the guidance of its parents, while the student attending university is usually older and therefore able to choose by himself the type of private educational investment he needs. In the latter case,

the role of the parent is just to decide the amount of transfer according to the own liquidity possibility

Nordblom (2003) shows that an increase in public schooling is not an effective policy to decrease the inequality between the education of rich and poor students. She develops a model in which households are composed of one altruistic parent and his child. The education level of the child positively depends on public schooling, parental investment and parental education (interpreted as accumulated of human capital). Parental investment includes expenditure in private education of the child (as additional books, computers, language courses and private teaching) and a monetary transfer (bequest). In this model, public and private education are assumed to be complements: an increase in public investment raises the marginal productivity of parental private investment. Nordblom defines the rich and the poor households in the following way: the rich one privately invests in the education of his child and leaves a bequest to him, while the poor one privately invests without leaving bequest. This model analyzes a highly stylized economy composed of one rich and one poor household. The level of education inequality is measured by the ratio between the human capital of the poor and the rich student.

Nordblom finds that an increase in public investment has an ambiguous effect on the educational gap. This ambiguity arises from the behavior of poor parents. In fact, further public expenditure makes the child richer with the result that the poor parent may substitute private investment with their own consumption. Because of this reaction, a policy increasing public spending could imply that the human capital of the rich student increases more than the human capital of the poor one.

We borrow the index of inequality from Nordblom and assume that the rich child receives a positive bequest from the parent while the poor one does not receive any direct monetary transfer. According to this definition, the poor student cannot invest either in private consumption or in private education therefore his education just depends on the public investment. We find that further public investment unambiguously increases the human capital of the poor student more than the human capital of the rich one. This result holds under the assumptions of both complementarity and substitutability between private and public education. When the public and private education are substitutes or weak complements then, the rich parent reacts to higher public investment by decreasing the bequest, and as consequence of this reduction the rich child reduces the investment in private education.

The main message of our paper can be the following: increasing public investment in the higher education system is an effective policy for equality under the assumptions of both complementarity and substitutability between private and public education. This theoretical result is in line with the empirical studies which confirm the substitutability between public and private education. This paper provides a justification for all the policies which promote further public investment in higher education system.

The rest of the paper is organized as follows: section 2 presents the general structure of the model. Section 3 defines the equilibrium strategy. In Section

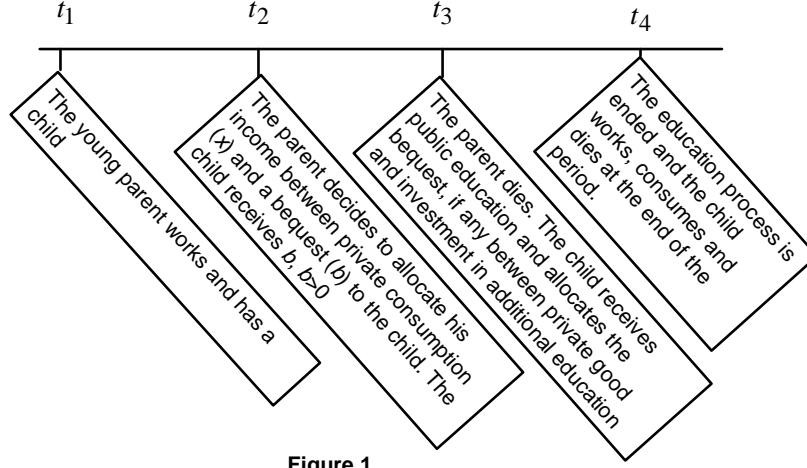


Figure 1
The sequence of events

4, we see if increasing public investment is an effective policy to increase the equality between the educational attainment of the rich and the poor student. Section 5 presents the main conclusions.

2 The Model

We consider an economy in which each household is composed of an altruistic parent and his child. The parent is altruistic in the sense that he includes the utility of the child in his own utility function. The sequence of the events is presented in Figure 1.

At time t_1 , the parent is young, works and has a son. At time t_2 , he decides to allocate his income between private consumption and a bequest to child. The bequest is the only way parents affect the educational attainment of his son. At period t_3 , the parent dies, in the meantime the child receives public education and allocates all entire bequest between private consumption and investment in additional education.

At time t_4 , the child enters the labour market. We consider a highly stylized labour market in which the child immediately gets a job once his educational process ends. The child consumes all his money earned in the labour market. We rule out any opportunity of saving because at the end of period t_4 the child dies and does not have son to leave the bequest to.

The private education is to be interpreted as an investment in time and private goods which increase the individual human capital. Therefore, the child can simultaneously receive both private and (free) public education.

The objective of the model is to study how a policy increasing public investment in higher education system affects the gap between the educational attainment of the rich and the poor student .

2.1 The child

The child is born at time t_1 . He does not take any decisions until time t_3 , when the parent dies. In fact, at time t_2 , he receives the bequest, if any, but does not use it before time t_3 . At time t_3 , he uses all his bequest to consume a private good or invest in private education. When the child decides the allocation of his bequest he takes into account the level of free public education currently provided by the public system. The investment in private education plays a focal role in the model because, together with the level of public schooling, it determines the amount of money the child will earn once he enters the labour market. At time t_3 , the human capital of the child h depends on the level of investment in public school E , and investment in private education I , according to the following production function¹:

$$h = f(I, E). \quad (1)$$

Assumption 1 The level of public investment is equal for all children in the economy.

Assumption 2 The production function $f(\cdot)$ is twice continuously differentiable, increasing and concave in I and E , that is $f_I, f_E > 0$ and $f_{II}, f_{EE} < 0$.

Assumption 2 explains that the human capital positively depends on private and public education. The sign of f_{IE} and f_{EI} , which are the cross derivatives of the production function, strongly determines the results of the paper: if this sign is negative (positive) the marginal productivity of each input is decreasing (increasing) in the other one.

In the rest of the paper we study how an increase in public schooling affects the gap between the educational attainment of the rich and the poor student when $f_{IE} > 0$ and $f_{IE} \leq 0$.

At time t_3 , the child can acquire the private good by using part of his bequest, which of course implies less investment in private education and consequently a lower level of human capital *ceteris paribus*.

At the end of time t_3 , the educational process ends, and at time t_4 the child enters the labour market. We assume that labour income ω is proportional to the level of human capital, according to the following relationship:

¹We assume neither the presence of innate ability nor correlation between a parent's human capital and his child's. See (Nordblom 2003) for the correlation between a parent's human capital and his child's.

$$\omega = f(I, E). \quad (2)$$

We normalize the wage to 1.

During the time t_4 , the child consumes all his labour income without saving anything.

Now consider the maximization problem faced by the child at time t_3 . He decides the level of private investment in education and consumption of private good, given the value of his bequest b . When he takes his decision, the child perfectly forecasts that the labour income he will receive at time t_4 depends on his current choices about the educational process and his bequest.

At time t_3 , utility function of the child is composed of the current utility of private good z , and the discounted utility of future labour income. This function is defined as follows:

$$U(z, \omega) = v(z) + \delta u(\omega). \quad (3)$$

Assumption 4 The functions $v(z)$ and $u(\omega)$ are twice continuously differentiable and strictly concave.

The budget constraint for the child is:

$$z + p_I I = b \quad (4)$$

where p_I is the price of investment in private education, and the price of private good is normalized to 1. Constraint (4) implies that, at time t_3 , the child spends all his bequest.

2.2 The parent

At the beginning of time t_2 , the parent decides how to allocate his income between current private consumption and a bequest for the child. The parent is characterized by the parameter γ that measures his degree of altruism. The utility function of the parent at time t_2 is composed of the utility of private good x , and the utility of the child. It is defined as follows:

$$u_p(x) + \gamma [U(z(b, E), \omega(b, E))], \quad (5)$$

where the subscript p denotes the parent.

Assumption 5 The function $u_p(x)$ is strictly concave and twice differentiable.

If the parent were not altruistic ($\gamma = 0$), then he would maximize just the utility of his current consumption of private good. For the rest of the paper we consider the case $\gamma > 0$.

At time t_2 , the parent's budget constraint is the following:

$$x + b = Y \tag{6}$$

The constraint (6) implies that the parent allocates all his income. The only form of saving for the parent is the bequest.

3 The Equilibrium of the Game

In this section we characterize the sub-game equilibrium of the model by solving backwards. We start from time t_3 by solving the maximization problem of the child, for a given value of his bequest. Then, we proceed by solving the maximization problem of the parent.

3.1 Third period: the child

Inserting (2) and (4) into the utility function (3) we obtain the child's maximization problem:

$$\max_I v(b - p_I I) + \delta u(f(I, E)). \tag{7}$$

The first order condition with respect to I is the following:

$$\delta u'(f(I, E)) f_I(.) = p_I v'(b - p_I I), \tag{8}$$

where $v'(.), u'(.),$ denotes respectively the marginal utility of private good z and future consumption. The factor $f_I(.)$ is the marginal productivity of investment in private education.

The first order condition shows that at optimum the marginal cost of an increase in I (in terms of reduction of z) equals the marginal benefit of such increase (in terms of higher future consumption).

The optimal level of private education decided by the child at time t_3 is defined as follows.

Definition 1 *The level of investment in private education, $I^*(b, E)$, satisfying the first order condition (8) is defined by the following expression:*

$$I^*(b, E) = \arg \max_I v(b - p_I I) + \delta u(f(I, E)). \quad (9)$$

The optimal level of private education depends on the bequest and the public education. After an exercise of comparative static on the first order condition (8) we obtain the following results:

i) If public and private investment in education are substitutes, then an increase in E implies a decrease in the optimal level of investment in private education. We totally differentiate the first order condition (8) and obtain:

$$\frac{dI^*(b, E)}{dE} = - \frac{\delta(u''(\cdot) f(\cdot)_E f(\cdot)_I + f_{IE}(\cdot) u'(\cdot))}{\delta(u''(\cdot) (f_I(\cdot))^2 + f_{II}(\cdot) u'(\cdot)) + (p_I)^2 v''(\cdot)}, \quad (10)$$

If $f_{IE}(\cdot) \leq 0$, then we have that $\frac{dI^*(b, E)}{dE} < 0$. When public and private investment in education are substitutes, the child reacts to an increase in public spending by unambiguously decreasing the level of investment in additional education.

If $f_{IE}(\cdot) > 0$, then the sign of $\frac{dI^*(b, E)}{dE}$ is ambiguous. When further public investment strongly increases the marginal productivity of the private investment, then the child can react by increasing the spending in private education.

ii) An increase in b implies an increase in the optimal level of investment in private education. By the total differentiation of the first order condition (8), we obtain:

$$\frac{dI^*(b, E)}{db} = - \frac{-p_I v''(\cdot)}{\delta(u''(\cdot) (f_I(\cdot))^2 + f_{II}(\cdot) u'(\cdot)) + (p_I)^2 v''(\cdot)} > 0. \quad (11)$$

By looking at the result of comparative static and treating b and E as exogenous parameter, we see that, the function $I^*(b, E)$ is increasing in the bequest and decreasing in the public education. These findings show that if there is an increase in public investment in the higher education system, then the optimal response of the child receiving positive bequest is to substitute away from the investment in private education to increase the consumption of the private good.

3.2 Second period: the parent

Substitute (6), and the child's utility as in (7) into (5). At time t_2 , the maximization problem faced by the parent is now:

$$\max_b u_p(Y - b) + \gamma [v(b - p_I I^*(b, E)) + \delta u(f(I^*(b, E), E))], \quad (12)$$

$$s.t. \ b \geq 0. \quad (13)$$

The constraint (13) rules out the possibility to leave debt to the child. The first order condition with respect to b is the following:

$$\frac{\partial U_p}{\partial b} = -u'_p(\cdot) + \gamma \left(v'(\cdot) \left(1 - p_I \frac{\partial I^*(\cdot)}{\partial b} \right) + \delta u'(\cdot) f_I(\cdot) \frac{\partial I^*(\cdot)}{\partial b} \right) \leq 0, \quad (14)$$

$$b \geq 0, \quad (15)$$

$$\frac{\partial U_p}{\partial b} b = 0, \quad (16)$$

where $u'_p(\cdot)$ denotes the marginal utility of the parent with respect to private good x . By (8) and the envelop theorem, (14) becomes:

$$\frac{\partial U_p}{\partial b} = -u'_p(Y - b) + \gamma v'(b - p_I I^*(b, E)) \leq 0, \quad (17)$$

The first order condition (17) implies that at the interior solution the marginal benefit of an increase in b (in terms of higher consumption of private good by the child) equals its marginal cost (in terms of a reduction of the private good by the parent). In equilibrium, an altruistic parent in interior solution uses his income both to consume private good and to leave a positive bequest. He increases the transfer to the point where its marginal benefit equals its marginal cost. The parent in corner solution does not leave anything to the child.

Let $\hat{b}(E)$ be the chosen level of bequest of a parent to his son, given that: *i*) the son will optimally choose I in stage 2, and *ii*) public investment is E .

With the following propositions we show how the level of bequest dependent on the income household.

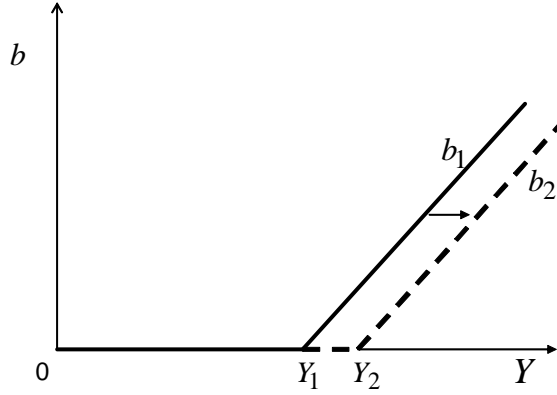


Figure 2
The optimal level of
the bequest

Proposition 2 *The highest income level such that a household is liquidity constrained increases with E if $f_{IE}(\cdot) \leq 0$, while it presents an ambiguous trend for $f_{IE}(\cdot) > 0$.*

Proof. See Appendix. ■

Proposition 2 says that if public and private investment are substitutes, then an increase in the public spending rises the highest income level such that the household chooses a bequest equal to zero. Proposition 3 explains how the optimal level of the bequest for the not liquidity constrained households depends on the income.

Proposition 3 *The optimal level of the bequest chosen by the not liquidity constrained households is increasing in the income.*

Proof. See Appendix. ■

Proposition 3 says that for the not liquidity constrained households higher income level implies higher bequest. We represent this relationship in figure 2.

The increasing part of the function b_1 represents the optimal level of bequest for each level of income higher than the highest level (Y_1) such that the household are liquidity constrained. In this case we are assuming the public and private education are substitutes. Now consider an increase in the public spending such that the income threshold shifts from Y_1 to Y_2 . As figure 2 illustrates, higher public investment shifts the bequest function on the right (toward b_2). The households with an income between Y_1 and Y_2 become liquidity constrained just

once the increase in public investment has occurred. In the absence of public policy these households have decided to leave a relatively low level of bequest, therefore even a small increase in public investment is enough to convince them to remove the transfer to the child. The households which, in the absence of policy, have income higher than Y_2 , continue to leave a positive bequest to the child even after the public investment has increased. For these households the rise in public spending is not so strong to motivate a removal of the bequest.

Finally, the households with an income lower than Y_1 are liquidity constrained before and after the increase in public investment. Children from these households always benefit from higher public spending because the only way they have to improve their level of education is the public policy.

In the next section we focus on the relationship between private investment in human capital and public education, and study the effect of an exogenous increase in public schooling on the equilibrium strategies.

4 Public education and human capital

Following Nordblom (2003), we assume a very simple economy composed of two households: one is liquidity constrained and the other is not. We refer to the former as the rich household and to the latter as the poor one.

The poor and the rich child have different levels of human capital because, at time t_2 , the poor one does not receive a bequest and can neither invest in private education nor consume private good.

4.1 Inequality and human capital

We borrow the inequality index from Nordblom (2003) and assume the case of the altruistic parent and thrifty child ($\delta > 0$). The inequality index is defined as follows:

Definition 4 *The gap between the educational attainment of the rich and poor child is defined by the following index:*

$$G(E) = \frac{f(0, E)}{f\left(I^*\left(\widehat{b}(E), E\right), E\right)}. \quad (18)$$

Now, let us assume that there is an exogenous increase in public investment. Such more public investment is free for the households. Furthermore, to rule out the scenario in which all the households are liquidity constrained we assume that the income of the rich household is sufficiently higher than the highest income level such that the household leaves no bequest.

The effect of such policy on the inequality can be obtained by deriving the index in (18) with respect to E .

Proposition 5 *If the public and private investment in education are substitutes or the marginal productivity of each one does not depend on the other, then increasing public investment decreases the level of educational gap.*

Proof of Proposition 5

Since we are analyzing the case of substitutability between public and private investments, then for the rest of the proof we assume $f_{IE}(I, E) < 0$.

Firstly, we state the condition for increased public education to decrease educational gap as follows:

$$\frac{\partial G(E)}{\partial E} > 0. \quad (19)$$

Once calculated the partial derivative of the index (18), we rewrite the condition (19) as follows:

$$\frac{1}{f(.)^2} \left[f_E(0, E) f(I^*(.), E) - \frac{df(I^*(.), E)}{dE} f(0, E) \right] > 0^2. \quad (20)$$

Given $\frac{f(I^*(.), E)}{f(0, E)} > 1$, after simple algebra, we show that the sufficient condition for (20) to hold is the following:

$$f_E(0, E) > \frac{df(I^*(.), E)}{dE}. \quad (21)$$

Given $\frac{df(I^*(.), E)}{dE} = f_{I^*}(I^*(.), E) \frac{dI^*(\hat{b}(E), E)}{dE} + f_E(I^*(.), E)$, we rewrite the (21) as follows:

$$f_E(0, E) - f_E(I^*(.), E) > f_{I^*}(I^*(.), E) \frac{dI^*(\hat{b}(E), E)}{dE}.^3 \quad (22)$$

²Where $f_E(0, E)$ is the partial derivative of the production function $f(0, E)$ with respect to E , and $\frac{df(I, E)}{dE}$ is the total derivative of the function $f(I, E)$ with respect to E .

³We recall that $f_{I^*}(I^*(.), E)$ and $f_E(I^*(.), E)$, are respectively the partial derivative of the production function $f(I^*(.), E)$ with respect to I^* and E .

Lemma 6 For every $E > 0$ and $I^* > 0$, we have⁴:

$$\frac{d\hat{b}(E)}{dE} = \frac{\left[p_I \gamma v''(\cdot) \frac{-[\delta u''(\cdot) f_{I^*}(\cdot) f_E(\cdot) + \delta u' f_{I^* E}(\cdot)]}{[\delta u''(\cdot) (f_{I^*}(\cdot))^2 + (p_I)^2 v''(\cdot) + \delta u' f_{I^* I^*}(\cdot)]} \right]}{\left[u_p''(\cdot) + \gamma v''(\cdot) \frac{\delta u''(\cdot) (f_{I^*}(\cdot))^2 + \delta u' f_{I^* I^*}(\cdot)}{[\delta u''(\cdot) (f_{I^*}(\cdot))^2 + (p_I)^2 v''(\cdot) + \delta u' f_{I^* I^*}(\cdot)]} \right]} \quad (23)$$

$$\frac{dI^*(\hat{b}(E), E)}{dE} = \frac{-[\delta u''(\cdot) f_{I^*}(\cdot) f_E(\cdot) + \delta u' f_{I^* E}(\cdot)] + p_I v''(\cdot) \frac{d\hat{b}(E)}{dE}}{\delta u''(\cdot) (f_{I^*}(\cdot))^2 + (p_I)^2 v''(\cdot) + \delta u' f_{I^* I^*}(\cdot)}. \quad (24)$$

For $f_{IE}(I, E) < 0$ and by lemma 6, we have the following results: i) the left hand side of (22) is positive, and ii) given $\frac{d\hat{b}(E)}{dE} < 0$, then $\frac{dI^*(\hat{b}(E), E)}{dE} < 0$

Since, part ii) implies that the right hand side is negative, then the condition (22) holds.

For $f_{IE}(I, E) = 0$ and by Lemma 6, we have the following results: i) the left hand side is zero, and ii) given $\frac{d\hat{b}(E)}{dE} < 0$, then $\frac{dI^*(\hat{b}(E), E)}{dE} < 0$. Hence, even in this case the condition (22) holds.

■

Proposition 5 shows that if public and private education are substitutes ($f_{IE}(I, E) < 0$) or the marginal productivity of each input does not depend on the other ($f_{IE}(I, E) = 0$), then an increasing public investment is an effective policy to increase the equality between the educational attainment of the rich and the poor student.

Consider the case $f_{IE}(I, E) < 0$. Higher public investment has a positive effect on the human capital of both the rich and the poor child. However, the human capital of the poor student increases more than the human capital of the rich one. The reason for this result is explained by two mechanisms working on condition (22). Firstly, the rich parent reacts to the increase in public investment by decreasing the bequest and, as consequence of this reduction, the rich child reduces his private investment in education. Secondly, the positive direct effect of public education on human capital is stronger for the poor student than for the rich one.

So far we have shown that if public and private education are substitutes and the investment in private education is decided by the child, then more public investments unambiguously increase the level of equality between the human capital of the rich and the poor student.

Now consider $f_{IE}(I, E) = 0$. In this case the marginal productivity of public (private) investment does not depend on the private (public) one. In this case the direct effect of increasing public investment on the human capital is the same for both the rich and the poor student. Hence, the reduction of private

⁴Where: $\frac{dI^*(\hat{b}(E), E)}{dE} = \frac{\partial I^*(\hat{b}(E), E)}{\partial E} + \frac{\partial I^*(\hat{b}(E), E)}{\partial \hat{b}} \frac{d\hat{b}(E)}{dE}$. See the Appendix for the proof of the Lemma

education by the rich child is the main reason for the human capital of the poor student to increase more than human capital of the rich one.

Nordblom (2003) finds that an increase in public schooling has an ambiguous effect on the educational gap. She develops a model in which public and private education are complements and the private investment in education is decided by the parent.

We show that even for a low degree of complementarity higher public investment unambiguously decreases the educational gap between poor and rich students.

Proposition 7 *If public and private education are weakly complements, then higher public investment still implies a decrease in the educational gap.*

Proof. See Appendix. ■

Proposition 7 says that further public investment implies that the human capital of the poor student increases more than the human capital of the rich one, even for a low degree of complementarity. To see that, consider condition (22). For a low degree of complementarity the rich parent still reacts to increasing public investment by decreasing the bequest. This reduction still implies that the rich child reduces his investment in private education.

Hence, the positive direct effect a further public investment has on the human capital of the rich child is compensated by the negative indirect effect arising from the reduction of the private investment.

5 Concluding remarks

Empirical evidence shows that children whose parents are characterized by high level of income and high human capital are more likely to be enrolled at university and also have better higher education attainments.

This paper shows that a policy increasing public investment in the higher education system effectively decreases the educational gap between the poor and the rich student. This conclusion holds in a model in which the decision makers are both the parent and the child. The altruistic parent as first mover allocates his income between a monetary transfer for the child and private own consumption. In the successive period the child receives public education and decides how to spend the transfer between private consumption and investment in private education. We allow the child to benefit from both private and public education because the private one includes time and goods invested increasing the human capital.

We assume that the rich child receives a positive transfer by the parent while the poor one does not receive any transfer and therefore his education just depends on the public investment. Following Nordblom (2003) we model an economy composed of one rich and one poor household and measure the

educational inequality as the ratio between the educational achievement of the poor and the rich student.

The determinant of our finding is the behavior of the rich parent and his child. In fact, once the public spending has increased, the rich parents react by decreasing the transfer. Because of this reduction, the rich child decreases the investment in private education. This lower private investment implies that the educational level of the poor student increases more than the level of the rich one.

The qualitative results of our paper hold under the assumptions of both substitutability and weak complementarity between private and public education.

The presence of two decision-makers and the validity of our result even under complementarity differentiate our model from the previous literature. We are aware that a model with two decision-makers in which the private investment is chosen by the child is more appropriate for representing the educational process working at the university level.

Nordblom (2003) is the most recent theoretical work studying the impact of increasing public investment on the educational gap. She shows that if the public and private investment are complements and the latter is only decided by an altruistic parent, then higher public spending has an ambiguous effect on the education inequality. This is because once the public investment has increased, the income effect makes the child richer with the consequence that the poor parent may substitute private investment with personal consumption.

Our conclusions show that the degree of complementarity between public and private education could not be the main determinant of the effect the policy increasing public investment has on the educational gap in higher education system.

Further theoretical research should analyze the effectiveness of such policy by constructing a general equilibrium model and to introduce a tax system to finance the increasing level of public spending. The fact that the investment in private education is chosen by the student clearly determines the main result of the paper; further empirical research should investigate the magnitude of this phenomenon.

APPENDIX

Proof of Proposition (2)

Let us denote the foc (17) as $F(b, E, Y)$ such that:

$$F(b, E, Y) = -u'_p(Y - b) + \gamma v'(b - p_I I^*(b, E)), \quad (25)$$

by totally differentiation the (25), we obtain:

$$\frac{\partial F(\cdot)}{\partial b} db + \frac{\partial F(\cdot)}{\partial E} dE + \frac{\partial F(\cdot)}{\partial Y} dY = 0. \quad (26)$$

Since we are computing the level highest income such that the households are liquidity constrained and chose zero bequest, then we have $db = 0$.

Finally, we have:

$$\frac{dY}{dE} = -\frac{p_I \gamma v''(\cdot) \frac{dI^*(b, E)}{dE}}{u''_p(\cdot)}. \quad (27)$$

Given (10), we know that the following results hold: *i)* if $f_{IE}(\cdot) \leq 0$, then $\frac{dY}{dE} > 0$, *ii)* if $f_{IE}(\cdot) > 0$, then the sign of $\frac{dY}{dE}$ is ambiguous.

■

Proof of Proposition 3.

We proceed as in the proof of Proposition 2 and obtain:

$$b'(Y) = -\frac{\frac{\partial F}{\partial Y}}{\frac{\partial F}{\partial b}} = -\frac{u''_p(\cdot) + \gamma v''(\cdot) \left(1 - p_I \frac{dI^*(b, E)}{db}\right)}{-u''_p(\cdot)} > 0, \quad (28)$$

because, given the value of $\frac{dI^*(b, E)}{db}$ in (11), we have:

$$1 - p_I \frac{dI^*(b, E)}{db} = \frac{\left(u''(f_I(\cdot))^2 + u' f_{II}\right)}{\delta \left(v''(\cdot) p_I^2 + \delta u''(\cdot) (f_I(\cdot))^2 + \delta u' f_{II}\right)} > 0. \quad (29)$$

■

Proof of Lemma 6.

Since we know that $I^*(.)$ directly and through $\widehat{b}(E)$ depends on E , we need to find the value of $\frac{d\widehat{b}(E)}{dE}$. To do that, we rewrite the first order condition of the rich parent (17) as follows:

$$-u'_p \left(Y - \widehat{b}(E) \right) + \gamma v' \left(\widehat{b}(E) - p_I I^* \left(\widehat{b}(E), E \right) \right) = 0, \quad (30)$$

we derive with respect to E , and obtain:

$$\frac{d\widehat{b}(E)}{dE} = \frac{p_I \gamma v''(.) \frac{\partial I^*}{\partial E}}{u''_p(.) + \gamma v''(.) \left(1 - p_I \frac{\partial I^*}{\partial b} \right)}. \quad (31)$$

To calculate the value of (31) we need the value of $\frac{\partial I^*(\widehat{b}(E), E)}{\partial E}$ and $\frac{\partial I^*(\widehat{b}(E), E)}{\partial b}$. To find them, we rewrite the first order condition of the child (eq.8) as follows:

$$\delta u' \left(f \left(I^* \left(\widehat{b}(E), E \right), E \right) \right) f_{I^*} \left(I^* \left(\widehat{b}(E), E \right), E \right) = p_I v' \left(\widehat{b}(E) - p_I I^* \left(\widehat{b}(E), E \right) \right), \quad (32)$$

and by differentiating with respect to E , we obtain:

$$\frac{dI \left(\widehat{b}(E), E \right)}{dE} = \frac{\partial I^* \left(\widehat{b}(E), E \right)}{\partial E} + \frac{\partial I^* \left(\widehat{b}(E), E \right)}{\partial b} \frac{d\widehat{b}(E)}{dE}, \quad (33)$$

where:

$$\frac{\partial I^* \left(\widehat{b}(E), E \right)}{\partial E} = \frac{-(\delta u''(.) f_{I^*}(.) f_E(.) + \delta u' f_{I^*E}(.))}{\left(\delta u''(.) (f_{I^*}(.))^2 + (p_I)^2 v''(.) + \delta u' f_{I^*I^*}(.) \right)}, \quad (34)$$

$$\frac{\partial I^* \left(\widehat{b}(E), E \right)}{\partial b} = \frac{p_I v''(.)}{\left(\delta u''(.) (f_{I^*}(.))^2 + (p_I)^2 v''(.) + \delta u' f_{I^*I^*}(.) \right)}. \quad (35)$$

Now, given (34) and (35), we define the value of (31) as follows:

$$\frac{\widehat{db}(E)}{dE} = \frac{\left[p_I \gamma v''(.) \frac{-[\delta u''(.) f_{I^*}(.) f_E(.) + \delta u' f_{I^* E}(.)]}{[\delta u''(.) (f_{I^*}(.))^2 + (p_I)^2 v''(.) + \delta u' f_{I^* I^*}(.)]} \right]}{\left[u_p''(.) + \gamma v''(.) \frac{\delta u''(.) (f_{I^*}(.))^2 + \delta u' f_{I^* I^*}(.)}{[\delta u''(.) (f_{I^*}(.))^2 + (p_I)^2 v''(.) + \delta u' f_{I^* I^*}(.)]} \right]}. \quad (36)$$

Once found (34), (35), and (36), we obtain:

$$\frac{dI^*(.)}{dE} = \frac{-[\delta u''(.) f_{I^*}(.) f_E(.) + \delta u' f_{I^* E}(.)] + p_I v''(.) \frac{\widehat{db}(E)}{dE}}{\delta u''(.) (f_{I^*}(.))^2 + (p_I)^2 v''(.) + \delta u' f_{I^* I^*}(.)}. \quad (37)$$

For $f_{IE}(.) \leq 0$, we have $\frac{\widehat{db}(E)}{dE} < 0$, therefore $\frac{dI^*(.)}{dE} < 0$.
■

Proof of Proposition 7.

Proposition 5 showed that the increase in public schooling decreases the educational gap for $f_{IE}(.) < 0$ and $f_{IE}(.) = 0$. Hence, there exists a value $\varepsilon > 0$ approaching zero and a degree of complementarity $f_{IE}(.)$, such that $0 < f_{IE}(.) \leq \varepsilon$, implying that further public investment still decreases the educational gap.
■

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